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Modeling Activated Sludge Systems As Balanced Aquariums

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Abstract

A new model of the activated sludge system is presented, based on the balanced aquarium ecosystem, one of the most famous models in ecology and one of the most similar ecological structures to the wastewater treatment system. The new model uses information from ASM No. 1 to create a biological oriented modeling system, which provides continuous results and a simple graphical representation of the evolution of autotrophs and heterotrophs, as main biological categories taking part in the treatment process, and ammonification and oxidation, as main biochemical processes of the wastewater treatment, throughout the entire treatment sequence, providing additional information for those interested in the community responsible for the biological treatment of wastewater. The new model is actually a very flexible framework, the system's elements and the information used for the construction of the differential equations being at the hand of the plant operators or the scientists studying the activated sludge system.

Keywords: *Activated sludge, mathematical model, balanced aquariums*

Notations

X_S – slowly biodegradable substrate
$X_{B,A}$ – autotrophic biomass
$X_{B,O}$ – oxidation
X_{ND} – particulate biodegradable organic nitrogen concentration
S_O – dissolved oxygen concentration
S_{NH} – soluble ammonia nitrogen concentration
S_{ALK} – total alkalinity
$K_{O,A}$ – oxygen half-saturation coefficient for autotrophic biomass
K_{NO} – nitrate half-saturation coefficient for the denitrifying heterotrophic biomass
K_X – half-saturation coefficient for the hydrolysis of slowly biodegradable substrate
μ_A – maximum specific growth rate of autotrophic biomass
b_A – specific decay rate coefficient for autotrophic biomass
Y_A – yield for autotrophic biomass
i_{XB} – mass of nitrogen per mass of COD in biomass
η_g– correction factor for μ_H under anoxic conditions
k_a – ammonification rate
$X_{B,H}$ – heterotrophic biomass
$X_{B,N}$ – ammonification
X_P – particulate products from biomass decay
S_S – readily biodegradable substrate
S_{NO} – nitrate nitrogen concentration
S_{ND} – soluble organic nitrogen concentration
$K_{O,H}$ – oxygen half-saturation coefficient for heterotrophic biomass
K_{NH} –ammonia half-saturation coefficient for the autotrophic biomass
K_S – half-saturation coefficient for the heterotrophic biomass
μ_H – maximum specific growth rate of heterotrophic biomass
b_H – specific decay rate coefficient for heterotrophic biomass
f_P – fraction of the biomass leading to particulate products
Y_H – yield for heterotrophic biomass
i_{XP} – mass of nitrogen per mass of COD in products from biomass
η_h– correction factor for hydrolysis under anoxic conditions
k_h – maximum specific hydrolysis rate

1. Introduction

Modeling of the activated sludge processes has a 50 years history at this point, starting with McKinney's model (1). The crystallization of the existing data on this type of modeling, made under the care of The International Water Association in the mid 80's, resulted in the development of state-of-art models (ASMs – (2, 3, 4, 5)), currently considered the most reliable for in situ application (6, 7, 8, 9) and for the development of recent date additional models regarding the process (10, 11, 12, 13, 14, 15, 16).

However, the very large majority of existing models, as well as the ASMs, is analyzing the system as an engineering structure, seeing its biological components as mechanical parts of the system, and focusing on physico-chemical flows inside the treatment installations (17). From this point of view, the construction of mathematical models starting from the biological components might help explaining some aspects of the activated sludge treatment process that were so far ignored by plant operators.

From the natural ecosystems similar to the activated sludge plant, the aquarium ecosystem is the most adequate, being basically a grey box structure, with known and controlled inputs and outputs, and with quantifiable relations between its components (18, 19). As a mathematical representation, it is usually depicted as an oriented numerical digraph, with elements balances extractable as differential equations, as proven by Cox (20), using Stella II software, which provided, with the help of a Runge-Kutta fixed-step procedure (21), a graphical output to systems' temporal evolution. Sîrbu (22) used Mathcad 14.0 to solve the Silver Springs ecosystem, with a much simpler informatic design for the Runge-Kutta integration, and similar graphical results, providing a convenient platform for such models, which will be used for the present study.

2. Methods

Mathematical information regarding the system was provided by ASM No.1 (2). ASM No.1 consists of 13 components (the particulate and soluble elements from the wastewater) and eight processes (metabolic processes of the implied biological components of the system and biochemical processes). They can be easily decomposed into four new components: autotrophs and heterotrophs, as the main biologic categories, and ammonification and oxidation, as biochemical processes. The relations between these new components and the components of the ASM No. 1 are depicted in figure 1.

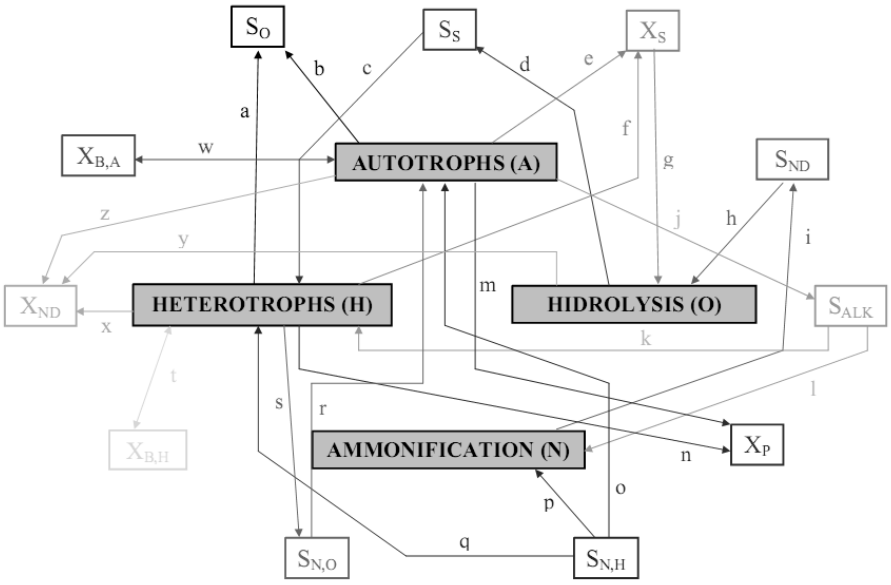

Figure 1. Relations between biological and biochemical components of the activated sludge system – grey boxes – and the components of the ASM No. 1 – colored boxes (a-w: system processes, as presented in Appendix C)

The complete equations of the four components are written by adding up the terms referring to each of them from the ASM No. 1 table and multiplying them with the

corresponding process rates from the same table, and they provide a single value for each moment of the process. Because of their complexity, some elements of the equation were encoded: switching functions (Appendix A), used for encoding process rates (Appendix B), which are then used for encoding process flux rates (Appendix C).The process flux rates are the ones used in the writing of the four equations of the new model's components, with the mention that ammonification and oxidation are considered dependent on a fraction of the activity of the first two components (heterotrophs and autotrophs), fractions represented by four coefficients (k_1 to k_4), which are to be empirically calculated for each treatment plant:

$$X_{B,H} = \left[p_1 \cdot \frac{(-15 \cdot iXB \cdot YH + 28 \cdot YH - 28)}{14 \cdot YH} + p_2 \cdot \frac{(-42.9 \cdot iXB \cdot YH + 53.04 \cdot YH - 53)}{40.04 \cdot YH} + p_4 \cdot (fp \cdot iXP - iXB) \right] [1]$$

$$X_{B,A} = \left[p_3 \cdot \frac{(-15 \cdot iXB \cdot YA + 28 \cdot YA - 65.98)}{14 \cdot YA} + p_5 \cdot (fp \cdot iXP - iXB) \right] \qquad [2]$$

$$X_{B,N} = k_1 \cdot X_{B,H} + k_2 \cdot X_{B,A} + \left(\frac{p_6}{14} \right) [3]$$

$$X_{B,O} = k_3 \cdot X_{B,H} + k_4 \cdot X_{B,A} \qquad [4]$$

The Runge-Kutta integration used by both Cox (20) and Sîrbu (22)for the balanced aquarium model provides linear results (being proposed as a restrained method for that reason by Cox himself), and is not suitable for a nonlinear, more realistic interpolation, the results being usable only as a prognosis tool. Therefore I used a spline interpolation (23) for the graphical representation of the real-time evolution of the activated sludge system. The model was constructed using Mathcad 14.0.

3. Results and Discussions

Because of the restrains of Mathcad, the notations slightly differ from the ones from the conceptual model, especially on the subscript text. Random datawas used for the demonstration of the way the model works.

Starting values were given for the four components of the system, as follows:

$XBH_1 := 120$

$XBA_1 := 1800$

$XBN_1 := 25$

$XBO_1 := 26$

Some of the system's coefficients are provided as single values, as required by ASM No.1, the values being again randomly generated:

$\mu h := 4.2 \quad \eta B := 1.2 \quad KOA := 0.6 \quad KNO := 1.9$

$\mu a := 3.1 \quad \eta H := 1.6 \quad KS := 1.5 \quad KNH := 1.6$

$bH := 0.9 \quad kA := 0.9 \quad KOH := 1.2 \quad KX := 0.1$

$bA := 0.7 \quad kN := 0.7$

The elements of the system that define the evolution of the four components were presented as strings of values corresponding to measurements made at every 30 minutes from the start of the process to 4 ½ hours from that point. The 4 ½ hours period is considered as a complete treatment cycle in some continuous treatment facilities, such as the one from Mohu, which processes the wastewater from Sibiu and the surrounding areas (24). Their values were also randomly generated:

$YA_j :=$	$YH_j :=$	$fp_j :=$	$iXB_j :=$	$iXP_j :=$	$SS_j :=$	$SO_j :=$	$SNO_j :=$	$SND_j :=$	$SNH_j :=$	$XND_j :=$	$XS_j :=$
15	40	0.8	0.8	2.1	19	8	8.5	2	10	30	15
14	80	0.6	0.6	2	19	7.7	8.4	1.98	9	28	14.9
18	166	0.7	0.5	1.8	19	7.5	8.45	1.9	8	27	14.75
15	27	0.5	1.1	1.2	17	7.3	8.3	1.7	6.5	25	14.5
8	5	0.4	0.2	1.2	13	7	8	1.5	5	20	14.35
27.6	5	0.8	1.2	1.4	10	6	7.8	1.45	4.8	17	14.24
27.6	8	0.9	1.3	1.3	8	5.8	7.7	1	4.75	16.5	14.01
−25	−6	0.7	1.9	1.4	6.6	5.7	7.56	0.425	4.75	16.5	13.99
−34	−5	0.6	1.8	0.3	3.4	5.5	7.5	0.4	4.65	16	13.89
−32	0	0.1	1.1	0.05	2.1	5	7	0.2	4.6	16	13.76

For each of the values from the strings, an equation system is generated, providing the starting values for the next 30 minutes period, after the following pattern (k_1 to k_4 coefficients were also randomly generated):

$$XBH_n := \left[p1_{\mathbf{n-1}} \cdot \frac{\left(-15 \cdot iXB_{n-1} \cdot YH_{n-1} + 28 YH_{n-1} - 28\right)}{14 YH_{n-1}} + p2_{n-1} \cdot \frac{\left(-42.9 \cdot iXB_{n-1} \cdot YH_{n-1} + 53.04 \cdot YH_{n-1} - 53\right)}{40.04 YH_{n-1}} + p4_{n-1} \cdot \left(fp_{n-1} \cdot iXP_{n-1} - iXB_{n-1}\right) \right]$$

$$XBA_n := \left[p3_{\mathbf{n-1}} \cdot \frac{-15 \cdot iXB_{n-1} \cdot YA_{n-1} + 28 YA_{n-1} - 65.98}{14 YA_{n-1}} + p5_{n-1} \cdot \left(fp_{n-1} \cdot iXP_{n-1} - iXB_{n-1}\right) \right]$$

$$XBN_n := 0.3 \cdot \mathbf{XBH}_n + 0.4 \cdot XBA_n + \left(\frac{p6_1}{14} \right)$$

$$XBO_n := 0.1 \cdot \mathbf{XBH}_n + 0.7 \cdot XBA_n$$

The resulting nine values for each of the components are added to the initial values of the system, resulting in four data strings which reflect the temporal evolution of the system throughout the treatment process (Het – heterotrophs, Aut – autotrophs, Amo – ammonification, Oxy - oxidation):

The four data strings were interpolated, resulting in a continuous image of the four components, as depicted in figure 2.

The resulting graphical representation differs from the usual representations provided by activated sludge models by at least two characteristics: i) it is a continuous representation of the biological elements of the system throughout the treatment sequence; ii) its purpose is not to provide proper functioning intervals for the elements of the system or intervention patterns for the plant operators, but to explain the evolution of the main categories of organisms that take part in the treatment process.

Het_j =
120
564.912
$3.033 \cdot 10^3$
$1.807 \cdot 10^4$
$3.757 \cdot 10^4$
$1.938 \cdot 10^5$
$1.457 \cdot 10^5$
$1.156 \cdot 10^5$
$8.477 \cdot 10^4$
$4.241 \cdot 10^4$

Aut_j =
$1.8 \cdot 10^3$
$4.817 \cdot 10^3$
$1.403 \cdot 10^4$
$4.78 \cdot 10^4$
$3.901 \cdot 10^4$
$1.086 \cdot 10^5$
$1.187 \cdot 10^5$
$9.805 \cdot 10^4$
$6.634 \cdot 10^4$
$2.009 \cdot 10^4$

Amo_j =
25
$2.112 \cdot 10^3$
$6.592 \cdot 10^3$
$2.491 \cdot 10^4$
$2.885 \cdot 10^4$
$1.052 \cdot 10^5$
$1.093 \cdot 10^5$
$8.326 \cdot 10^4$
$5.513 \cdot 10^4$
$2.294 \cdot 10^4$

Oxy_j =
26
$3.428 \cdot 10^3$
$1.012 \cdot 10^4$
$3.527 \cdot 10^4$
$3.106 \cdot 10^4$
$9.541 \cdot 10^4$
$9.766 \cdot 10^4$
$8.019 \cdot 10^4$
$5.491 \cdot 10^4$
$1.83 \cdot 10^4$

From that point of view, the model, although not as useful for the wastewater treatment process as the ASMs or other engineering based models, offers a different picture on the treatment system and might provide additional information for plant operators or scientists studying the activated sludge. The model is just a framework, and its elements can be divided into several other groups (e.g. splitting the two main categories – autotrophs and heterotrophs –into the subdivisions needed or wanted). The information used for the construction of the system's equations is not strictly restrained to the one contained by ASM No. 1, and can also be extracted from any other model of the wastewater treatment, offering, therefore, a large flexibility and an even larger complexity of the results possible to be provided.

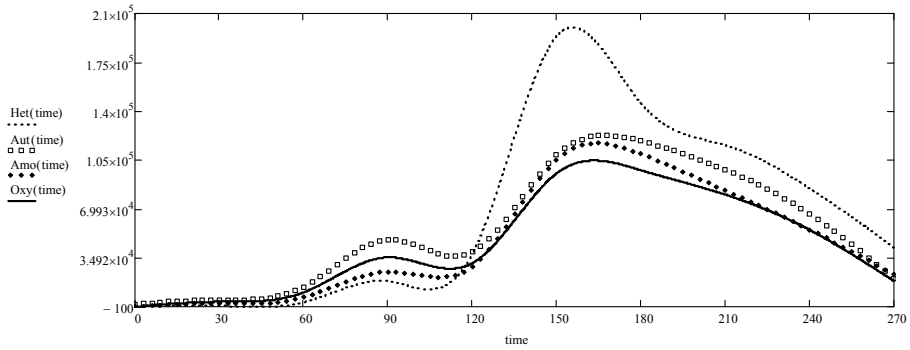

Figure 2. Trajectories of the four biological/biochemical components of the activated sludge system (spline interpolation; Het – heterotrophs, Aut – autotrophs, Amo – ammonification, Oxy - oxidation)

4. Conclusions

A new model of the activated sludge system was generated, based on one of the archetypal models of ecological systems: Odum's balanced aquarium.

The new model incorporates ASM No. 1 information into a biological oriented modeling system and provides continuous results for the evolution of autotrophs, heterotrophs, ammonification and oxidation throughout the treatment sequence, providing additional information for those interested in the community responsible for the biological treatment of wastewater.

The graphical output of the model is easily understandable and easy to use as prognosis tool for the evolution of the elements, and it is highly responsive to any modifications in the data constructing the system's equations.

The new model is actually a very flexible framework, the differentiation of the system's elements and the information used for the construction of the differential equations being at the hand of the plant operators or the scientists studying the activated sludge system.

Appendix A
Switching functions encoding

$$s_1 = \frac{S_S}{K_S + S_S} \quad \text{Eq. [A.1]} \qquad s_2 = \frac{S_O}{K_{O,H} + S_O} \quad \text{Eq. [A.2]}$$

$$s_3 = \frac{K_{O,H}}{K_{O,H} + S_O} \quad \text{Eq. [A.3]} \qquad s_4 = \frac{S_{N,O}}{K_{N,O} + S_{N,O}} \quad \text{Eq. [A.4]}$$

$$s_5 = \frac{S_{N,H}}{K_{N,H} + S_{N,H}} \quad \text{Eq. [A.5]} \qquad s_6 = \frac{S_O}{K_{O,A} + S_O} \quad \text{Eq. [A.6]}$$

$$s_7 = \frac{\frac{X_s}{X_{B,A}}}{\frac{X_s}{X_{B,A}} + K_X} \quad \text{Eq. [A.7]}$$

Appendix B
Process rates encoding

$$p_1 = \mu_H * s_1 * s_2 * X_{B,H} \quad \text{Eq. [B.1]}$$

$$p_2 = s_1 * s_3 * s_4 * \eta_g * X_{B,H} * \mu_H \quad \text{Eq. [B.2]}$$

$$p_3 = \mu_A * s_5 * s_6 * X_{B,A} \quad \text{Eq. [B.3]}$$

$$p_4 = b_H * X_{B,H} \quad \text{Eq. [B.4]}$$

$$p_5 = b_A * X_{B,A} \quad \text{Eq. [B.5]}$$

$$p_6 = k_a * S_{ND} * X_{B,H} \quad \text{Eq. [B.6]}$$

$$p_7 = k_n * s_7 * (s_4 + \eta_h * s_3 * s_4) * X_{B,H} \quad \text{Eq. [B.7]}$$

$$p_8 = p_7 * \frac{X_{ND}}{X_S} \quad \text{Eq. [B.8]}$$

Appendix C
Process flux rates

$$a = \frac{1 - Y_H}{Y} * p_1 \quad \text{Eq. [C.1]} \qquad b = \frac{4{,}57 - Y_A}{Y_A} * p_3 \quad \text{Eq. [C.2]}$$

$$c = \frac{1}{Y_H} * (p_1 + p_2) \quad \text{Eq. [C.3]} \qquad d = p_7 \quad \text{Eq. [C.4]}$$

$e = (1 - f_p) * p_5$	Eq. [C.5]	$f = (1 - f_p) * p_4$	Eq. [C.6]
$g = p_7$	Eq. [C.7]	$h = p_8$	Eq. [C.8]
$i = p_6$	Eq. [C.9]	$j = (\frac{i_{XB}}{14} - \frac{1}{7 * Y_A}) * p_3$	Eq. [C.10]
$k = \frac{1 - Y_H}{14 * 2{,}86 * Y_H} * p_2 - \frac{i_{XB}}{14}(p_1 + p_2)$	Eq. [C.11]	$l = \frac{1}{14} * p_6$	Eq. [C.12]
$m = f_p * p_5$	Eq. [C.13]	$n = f_p * p_4$	Eq. [C.14]
$o = (i_{XB} + \frac{1}{Y_A}) * p_3$	Eq. [C.15]	$p = p_6$	Eq. [C.16]
$q = i_{XB} * (p_1 + p_2)$	Eq. [C.17]	$r = \frac{1}{Y_A} * p_3$	Eq. [C.18]
$s = \frac{1 - Y_H}{2{,}86 * Y_H} * p_2$	Eq. [C.19]	$t = p_1 + p_2 + p_4$	Eq. [C.20]
$x = (i_{XB} - f_p * i_{XP}) * p_4$	Eq. [C.21]	$y = p_8$	Eq. [C.22]
$z = (i_{XB} - f_p * i_{XP}) * p_5$	Eq. [C.23]	$w = p_3 + p_5$	Eq. [C.24]

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